Breaking the electroweak symmetry and supersymmetry by a compact extra dimension

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(Received 10 July 2002; published 8 November 2002)

We reexamine in some more detail a recent specific proposal for the breaking of the electroweak symmetry and of supersymmetry by a compact extra dimension. Possible mass terms for the Higgs boson and the matter hypermultiplets are considered and their effects on the spectrum analyzed. Previous conclusions are reinforced and put on firmer ground.

DOI: 10.1103/PhysRevD.66.095003 PACS number(s): 12.60.Jv, 11.10.Kk, 11.15.Ex

I. INTRODUCTION AND MOTIVATION

Electroweak symmetry breaking (EWSB) remains an unsettled central problem in particle physics. No clear experimental signal has emerged yet which points towards a specific physical description of EWSB. The Higgs boson has not been found, nor any supersymmetric particle, which, as believed by many, could play a crucial role in triggering EWSB. Similarly any possible mechanism of dynamical EWSB, if realized, is at least well hidden in the relevant data so far. This is where we stand at the moment. It is very likely, on the other hand, that the progression of the upgraded Fermilab Tevatron runs and, especially, the coming in operation of CERN Large Hadron Collider (LHC) will change the situation in this decade, making available crucial data on the physics of EWSB. All this motivates further thoughts on this problem.

Where do we look for an orientation, however? Theoretically, the quadratic divergence of the Higgs boson mass in the standard model (SM) remains a crucial aspect of EWSB. This is neither new, however, nor sufficiently discriminating among alternative theoretical ideas. More significant, maybe, is the impressive series of electroweak precision tests (EWPT) performed in the 1990s mostly at the CERN e^+e^- collider LEP but also at the Tevatron and at SLAC Linear Collider SLC. As is well known, these data brilliantly confirm the SM at the level of the pure electroweak radiative corrections. Although indirectly, this suggests that any drastic departure from the SM cannot occur, if it does at all, below a few TeV. Furthermore, always indirectly, evidence emerges from the same data in favor of a light Higgs boson in the hundred GeV range. Here we stick to this interpretations of the EWPT, not unavoidable but plausible: there is indeed a light Higgs boson, close to the direct lower bound of about 110 GeV, while the physics remains perturbative up to 2–3 TeV energies at least.

This view would have a problem, however, if we also supposed at the same time that no new weakly interacting particles were present below the cutoff scale Λ, at or above 2–3 TeV. The dominant radiative correction to the Higgs boson mass, from the top loop, cutoff at Λ would in fact be (G_F is the Fermi constant and m_t the top mass):

$$\delta m_H^2(\text{top quark}) = \frac{3}{\sqrt{2}\pi^2} G_F m_t^2 \Lambda^2 = (0.9\ \ \text{TeV})^2 \left(\frac{\Lambda}{3\ \ \text{TeV}}\right)^2 \tag{1.1}$$

at least about 100 times bigger than the supposed physical Higgs squared mass. The usual hierarchy problem, coupled with the knowledge of the top mass, has acquired a ‘‘low energy’’ aspect. We underline that this was not the case, a decade ago, when the EWPT were not available and the top mass was not known. Given the special role of LEP in the EWPT, one can call this the ‘‘LEP paradox’’ [1].

All this sounds pretty familiar as an argument in favor of the existence of superpartners, and maybe it is. With the introduction of the loops of the top squark, of mass $m_{\tilde{t}}$, Eq. (1.1) turns into

$$\begin{aligned} \delta m_H^2(\text{top quark–top squark}) &= \frac{3}{\sqrt{2}\pi^2} G_F m_t^2 m_{\tilde{t}}^2 \log\frac{\Lambda^2}{m_{\tilde{t}}^2} \\ &= 0.1\ \ m_{\tilde{t}}^2 \log\frac{\Lambda^2}{m_{\tilde{t}}^2} \end{aligned} \tag{1.2}$$

still divergent, but only logarithmically. The top squark and the other superpartners might exist then, but where? It is in the very spirit of this entire argument that the correction (1.2) and the other contributions to m_H^2 should not exceed significantly the $(100\ \text{GeV})^2$ range without an accidental tuning among the different parameters involved. In turn, by inspection of definite supersymmetric extensions of the SM, this has led to the expectation that some superpartner, as the Higgs boson itself, had to be discovered, in particular at LEP, which did not happen. Since this was an expectation and not a theorem, the failure to find supersymmetry at LEP is not of immediate interpretation either. With some amount of tuning in the space of parameters, standard superpartners, with masses close to the current lower bounds, can certainly exist, ready to be found at the upgraded Tevatron and/or at LHC. The success of gauge coupling unification supports this view. On the bad side, however, by allowing an increasing amount of tuning, all superpartners could escape detection even at LHC.

All this justifies, in our opinion, the exploration of alternative possibilities: we seek a model with a naturally light Higgs boson in the 100 GeV region, perturbative up to a few TeV at least and with a structure in between possibly determined in terms of a minimum number of parameters. This has motivated the proposal of Ref. [2]. In this paper we return to it in some more detail, also in view of the contents of Refs. [3–6].

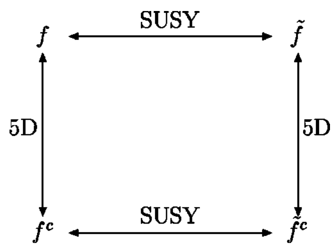

FIG. 1. Component diagram of the hypermultiplet in 5D.

The structure of the paper is the following. In Sec. II we recall the solution of the LEP paradox proposed in Ref. [2]. Section III contains a description of the complete extension of the SM and of the in principle relevant parameter space. This includes suitable mass terms for the matter and Higgs hypermultiplets as discussed in [6]. In Sec. IV we give a detailed discussion of EWSB with the inclusion of mass terms, small relative to $1/R$. The spectrum of the model and the consequent phenomenological implications are summarized in Sec. V.

II. A SOLUTION OF THE LEP PARADOX

We suppose that supersymmetry is relevant to solve the LEP paradox, as defined above, and that the left-handed top quark, with its doublet $Q=(t,b)_L$, and the right-handed top quark t_R live in 5 dimensions of coordinates (x_μ,y). For every matter Weyl spinor f this amounts to introducing a hypermultiplet of (x_μ,y)-dependent fields according to the scheme of Fig. 1, where f^c denotes a spinor with the same chirality of f but opposite quantum numbers. The 5th dimension is viewed as a segment of length $\pi R/2$ with Dirichlet $(+)$ or Neumann $(-)$ conditions on the two boundaries: $(+,+)$, $(-,-)$, $(+,-)$, $(-,+)$, respectively, for the fields f, f^c, $\tilde{f}$, $\tilde{f}^c$. This is a unique way, consistent with the symmetries of the free 5D Lagrangian, to obtain a spectrum with a single massless mode for the fermion f only. The spectrum for the entire hypermultiplet with these boundary conditions is given in Fig. 2. All fields are periodic over a circle of radius R. The 5th dimension can be viewed as compactified on a $S^1/(Z_2\times Z_2')$ orbifold where Z_2 and Z_2' are the reflections around the two boundaries at $y=0$ and $y=\pi R/2$. Supersymmetry is broken à la Scherk and Schwarz by the boundary conditions.

Consistently with supersymmetry, the top Yukawa coupling can be introduced as a superpotential term localized at one of the boundaries, say $y=0$,

$$\mathcal{L}_Y=\int \mathrm{d}y\ \delta(y)\int\ \mathrm{d}^2\theta\lambda_t\hat{h}\ \hat{Q}\ \hat{U}+\mathrm{H.c.} \tag{2.1}$$

where $\hat{h}$, $\hat{Q}$, $\hat{U}$ are $N=1$ chiral multiplets. In particular $\hat{Q}$ and $\hat{U}$ each contain the fields f and $\tilde{f}$ of Fig. 1 with the corresponding quantum numbers. It is irrelevant at this stage whether $\hat{h}$ does or does not have a y dependence. We assume that the scalar $\hat{h}$ contains a y independent component $h^0(x)$ which plays the role of the standard Higgs field.

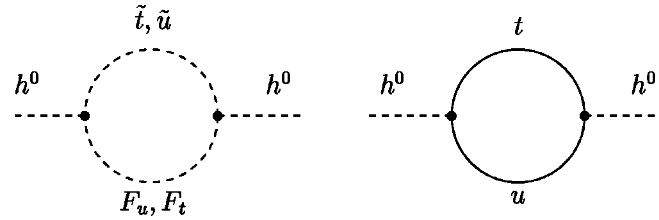

FIG. 3. One-loop diagrams contributing to the mass squared of the Higgs boson.

We are in a position to compute the one-loop contribution to the Higgs boson mass due to the coupling (2.1). This is most readily done by means of the propagators in mixed (p_μ,y) space $G_i(p;y,y')$ for the different components of the superfield, $i=f,\tilde{f},F$ [7]. Corresponding to the diagrams of Fig. 3 one has

$$\delta m_H^2=3\frac{\lambda_t^2}{2\pi R}\int\frac{\mathrm{d}^4p}{(2\ \pi)^4}\{-\mathrm{Tr}[G_t(p)G_u(p)]+G_{F_u}(p)G_{\tilde{t}}(p)+G_{F_t}(p)G_{\tilde{u}}(p)\} \tag{2.2}$$

where $G_i(p)=G_i(p;0,0)$. Using Eq. (B9) of Appendix B we obtain [2]

$$\delta m_H^2=-\frac{3\,\hat{y}_t^2}{16\,R^2}\int_0^\infty \mathrm{d}x\,x^3\left[\coth^2\left(\frac{\pi x}{2}\right)-\tanh^2\left(\frac{\pi x}{2}\right)\right]$$
$$=-\frac{63\zeta(3)}{8\pi^4}\frac{\hat{y}_t^2}{R^2} \tag{2.3}$$

where $\zeta(3)=1.20$ and $\hat{y}_t=\lambda_t/(2\pi R)^{3/2}$ is the top Yukawa coupling in 4D (anticipating a y-dependent Higgs field as well). The finiteness of Eq. (2.3) is a consequence of local supersymmetry conservation in 5D, as discussed below.

A. The relation between the compactification scale and the cutoff

The finiteness of Eq. (2.3) and the spectrum in Fig. 2, with all extra particles in the top hypermultiplet living at or above the compactification scale $1/R$, look as a right step in the direction of solving the LEP paradox. The price to be paid, however, is the non-renormalizability of the coupling (2.1) in 5D. Any model that incorporates the physics of Sec.

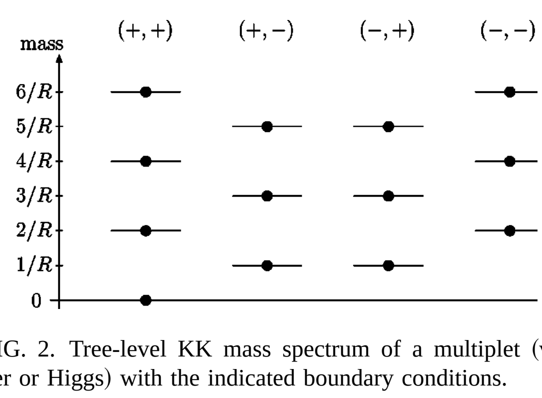

FIG. 2. Tree-level KK mass spectrum of a multiplet (vector, matter or Higgs) with the indicated boundary conditions.

II must be thought of as an effective field theory valid up to some cut-off scale Λ. This is not necessarily a problem, however, as long as Λ is itself not lower than 2–3 TeV and is sufficiently bigger than the compactification scale $1/R$ so that Eq. (2.3), or similar ones, remain quantitatively meaningful in the usual sense of effective field theories.

The relation between $1/R$ and Λ can be fixed by requiring that the top Yukawa coupling in Eq. (2.1) becomes nonperturbative at Λ, taking into account the increasing number of states whose thresholds are crossed at every unit of $1/R$. With this assumption, the value of $\hat{y}_t$ at Λ can either be estimated by means of usual dimensional arguments, properly adapted to 5D [8],

$$\hat{y}_t(\Lambda) \simeq \frac{1}{16\pi^2}\left(\frac{24\pi^3}{2\pi\Lambda R}\right)^{3/2} \simeq 8.2(\Lambda R)^{-3/2} \tag{2.4}$$

or by noticing that the expansion parameter in a 4D calculation involving the top Yukawa coupling is [20]

$$\frac{2\hat{y}_t^2}{16\pi^2}(N_{KK})^3$$

where $N_{KK} \simeq \Lambda R$ is the number of modes below Λ, hence

$$\hat{y}_t(\Lambda) \simeq \frac{4\pi}{\sqrt{2}}\left(\frac{1}{\Lambda R}\right)^{3/2} \simeq 8.9(\Lambda R)^{-3/2}. \tag{2.5}$$

Matching this value with the measured top Yukawa coupling at the weak scale gives $\Lambda R \simeq 5$ [2]. Note that $\hat{y}_t$ at Λ has not increased from 1 by more than 20% or so. The one-loop evolved $\hat{y}_t$ starts growing rapidly at $\Lambda \simeq 6/R$. From the 4D viewpoint, it is the multiplicity of states, rather than the increase of $\hat{y}_t$ itself, that causes the loss of perturbativity.

Is $\Lambda R \simeq 5$ enough to defend the predictivity of an equation such as (2.3)? We claim that it is, as it can be checked by writing the most general Lagrangian, involving the top and the Higgs fields, consistent with the various symmetries and containing operators of arbitrarily high dimensions, all assumed to saturate perturbation theory at Λ. The corrections that these extra couplings induce are not large. The value of Λ itself, or of $1/R$, will be set in the following. We also anticipate that the gauge couplings, growing more slowly than $\hat{y}_t$, remain perturbative below or at Λ.

III. THE COMPLETE MODEL AND THE RELEVANT PARAMETER SPACE

The most straightforward way to include the gauge and Higgs multiplets is to take every field in 5D. With matter and gauge fields in 5D, (discrete) momentum conservation holds in the 5th direction, thus weakening the lower bounds on $1/R$. Furthermore, it is essential that the Higgs boson also lives in 5D if one does not want to duplicate the Higgs multiplets as in standard supersymmetric models (see below).

The parity assignments of the fields in the gauge multiplet, a 4D vector A_μ, a 4D complex scalar $\varphi=(1/\sqrt{2})(\Sigma + iA_5)$ and two Weyl spinors $\tilde{\lambda}$, $\tilde{\lambda}^c$ for every generator of the gauge group $SU(3)\times SU(2)\times U(1)$ are fixed to be $(+,+)$, $(-,-)$, $(+,-)$ and $(-,+)$, respectively, by Lorentz, gauge and supersymmetry invariance in 5D. Hence only the vector zero mode is massless, as shown in Fig. 2.

TABLE I. Continuous R charges for gauge, Higgs boson and matter components. Here, m represents q, u, d, l, e.

R	gauge V	Higgs H	matter M
$+2$		h^c	
$+1$	$\tilde{\lambda}$	$\tilde{h}^c$	$\tilde{m}$, $\tilde{m}^c$
0	A^μ, A^c	h	m, m^c
-1	$\tilde{\lambda}^c$	$\tilde{h}$	

As to the Higgs hypermultiplet two choices are in principle possible for the parity assignments: the $(+,+)$ given to a fermionic component (as for the matter hypermultiplets) or to a scalar component, with the parities of all other fields fixed automatically. Only the second choice leads to a non-anomalous theory and leaves the zero mode of the Higgs field massless. The two Higgsinos, $(+,-)$ and $(-,+)$, are all paired to get a Dirac mass at $(2n+1)/R$, $n=0,1,\ldots$ (see Fig. 2).

A. Residual symmetries after the orbifold projection

The 5D supersymmetric gauge Lagrangian is fixed at this stage. The symmetries that survive the orbifold projection other than gauge and flavor symmetries are the following.

(1) 5D supersymmetry with y-dependent transformation parameters ξ_1, ξ_2 subject to the boundary conditions $(+,-)$ and $(-,+)$ [9]. This implicitly assumes the promotion of supersymmetry to a local symmetry, hence to supergravity. We note that the scale of the supergravity couplings need not be connected with the cut off Λ of Sec. II.

(2) A continuous R symmetry with R charges given in Table I, intact even after EWSB. The absence of any A terms or Majorana gaugino masses can be traced back to this symmetry.

(3) A local y parity under which any field transforms as

$$\varphi(y) \to \eta\, \varphi(\pi R/2 - y) \tag{3.1}$$

where η is the parity assignment at any one of the two boundaries. Note that this cannot be extended to a global 5D parity symmetry which includes the two boundaries since $Z_2 \times Z_2'$ is the most general discrete symmetry group on S^1 [9]. This symmetry is enough, however, to forbid local mass terms for the hypermultiplets.

These symmetries strongly constrain the form of the 5D (bulk) Lagrangian, $\mathcal{L}_5$, but leave open the possibility of suitable Lagrangian terms at the two boundaries, so that, for the total Lagrangian

$$\mathcal{L} = \mathcal{L}_5 + \delta(y)\mathcal{L}_4 + \delta\left(y - \frac{\pi R}{2}\right)\mathcal{L}_4'. \tag{3.2}$$

Some of the terms in $\mathcal{L}_4$ and $\mathcal{L}_4'$ will in fact be generated anyhow, subject only to the usual non-renormalization properties of supersymmetry. One important fact to notice about $\mathcal{L}_4$ and $\mathcal{L}_4'$ is that they respect different $N=1$ supersymmetries, associated with the parameters ξ_1 and ξ_2, which vanish, respectively, at $y=\pi R/2$ and $y=0$. In practice, to write down the most general $\mathcal{L}_4$ and $\mathcal{L}_4'$ one employs the usual rules of 4D $N=1$ supersymmetry after identification of the proper supermultiplets. In turn this identification can be done by considering the 5D supersymmetry transformation and setting there either ξ_1 or ξ_2 equal to zero. If one looks at the Higgs hypermultiplet it is immediate to see, anyhow, that the supermultiplets whose components have the same orbifold parities and do not vanish at the boundaries are $[h(+,+),\tilde{h}(+,-)]$ and $[h^\dagger(+,+),\tilde{h}^c(-,+)]$, respectively, at $y=0$ and $y=\pi R/2$. This is what makes it possible to write down Yukawa couplings both for up and for down quarks or for the leptons to a single Higgs field $h(+,+)$ and still be consistent with (local) supersymmetry. The Yukawa couplings for the up quarks are located at $y=0$ and the Yukawa couplings for the down quarks and the leptons at $y=\pi R/2$ [2].

Finally we note that $\mathcal{L}_4$ and $\mathcal{L}_4'$ can contain a Fayet-Iliopoulos term associated with the hypercharge $U(1)$. We shall come back to this possible term in Sec. III C.

B. Gauge anomalies and hypermultiplet mass terms

The boundary conditions, or the orbifolding, turn the vectorial 5D Lagrangian into a chiral theory. This is obviously the case in the pure gauge-matter sector since the orbifold projections select chiral fermionic zero modes. It is also true, however, in the gauge-Higgs sector in spite of parity conservation and of the Dirac nature of all Higgsino masses. Some of the Kaluza-Klein (KK) vector bosons couple to vector currents and some others to axial currents. Similarly some of the KK states of the gauge multiplet φ are scalars and some pseudoscalars. One wonders then if gauge anomalies may appear localized on the boundaries [5,10].

The naive answer to this question turns out to be correct. To ensure gauge invariance and the conservation of the corresponding 5D gauge current, it is enough that the fermionic zero modes, after the orbifold projection, satisfy the usual 4D anomaly cancellation condition [10]. Since the matter fermions are anomaly free and there are no massless Higgsinos, the orbifold construction described above is anomaly free. A qualification of this statement is necessary, however. Because of the Higgs sector, gauge invariance can be maintained at the quantum level, but not, at the same time, as the local parity symmetry defined in Sec. III A. In particular there is no regularization that preserves both symmetries [6,11].

The breaking of the local y parity makes it possible that there be mass terms for the hypermultiplets. For the hypermultiplet of components $(\psi,\psi^c,\varphi,\varphi^c)$, the 5D mass term consistent with the residual supersymmetry after the orbifold projection is [6]

$$\mathcal{L}_m=-[\psi m(y)\psi^c+\text{H.c.}]-M^2(|\varphi|^2+|\varphi^c|^2)$$
$$-2M[\delta(y)+\delta(y-\pi R/2)](|\varphi|^2-|\varphi^c|^2) \quad (3.3)$$

irrespective of the specific boundary conditions for the different components. Note the appearance of the boundary term. In the formulation of the theory on a circle S^1, the mass term $m(y)$ has to satisfy $(-,-)$ boundary conditions to be also $Z_2\times Z_2'$ invariant. Furthermore, bulk supersymmetry implies that $m(y)$ is piecewise constant in the four different patches of the circle; hence,

$$m(y)=M\,\eta(y),$$

$$\eta(y)=\begin{cases}+1, & y\in(0,\pi R/2)\cup(\pi R,3\pi R/2),\\ -1, & y\in(\pi R/2,\pi R)\cup(3\pi R/2,2\pi R).\end{cases} \quad (3.4)$$

The effect of a mass term like Eq. (3.3) on the spectrum is discussed in Sec. III D. We point out, however, that if these mass terms are vanishing at tree level, the non-renormalization theorems guarantee that they can only be renormalized by finite, negligibly small, non-local corrections associated to the orbifold breaking of global supersymmetry.

C. The Fayet-Iliopoulos term

It was pointed out in Ref. [3] that a Fayet-Iliopoulos (FI) term on the boundaries is induced in the model under examination by one-loop corrections involving the gauge coupling to the hypercharge Y. At first this is not surprising since a FI term in 4D is both gauge invariant and globally supersymmetric. It is, however, also somewhat worrisome, still in view of the 4D properties of a FI term. In 4D the FI term breaks supersymmetry and/or the gauge symmetry in the vacuum, something we would not like to happen in view of the previous discussion. Furthermore, it is not gauge invariant in supergravity if the $U(1)$ charge of the gravitino vanishes [12,13], which is the case for Y. Finally the one-loop FI term arises only in the presence of mixed $U(1)$ gravitational anomalies.

None of these unpleasant features necessarily survive in 5D [6]. In particular they are not shared by the FI term in the model under consideration, which takes the form

$$\mathcal{L}_\xi=\xi\left[\delta(y)(X_3-\partial_y\Sigma)+\delta\left(y-\frac{\pi R}{2}\right)(X_3+\partial_y\Sigma)\right] \quad (3.5)$$

where Σ is the real scalar in the vector hypermultiplet and X_a is the $SU(2)_R$ triplet of auxiliary fields of the 5D vector multiplet [14]. X_3 and Σ intervene in the quadratic Lagrangian without a mixed term

$$\mathcal{L}^{(2)}=\frac{1}{2}X_3^2+\frac{1}{2}(\partial_M\Sigma)(\partial^M\Sigma). \quad (3.6)$$

For the purposes of this paper, it is important to observe that, in the vacuum, from Eqs. (3.5),(3.6)

$$X_3 = -\xi\left[\delta(y) + \delta\left(y - \frac{\pi R}{2}\right)\right]$$
$$\partial_y \Sigma = -\xi\left[\delta(y) - \delta\left(y - \frac{\pi R}{2}\right)\right] \tag{3.7}$$

showing explicitly that the D-flatness conditions at both boundaries $D = X_3 - \partial_y \Sigma = 0$, $D' = X_3 + \partial_y \Sigma = 0$ are satisfied. Note that, on the S^1 circle, the vacuum form of Σ is $\langle \Sigma \rangle = -(\xi/2)\eta(y)$ with $\eta(y)$ as in Eq. (3.4). This amounts to a spontaneous breaking of the local y parity. In turn, after the replacement of Eq. (3.7) in the interaction terms of the X_3 and Σ fields with a generic hypermultiplet of hypercharge Y,

$$\mathcal{L}_{\text{int}} = g_Y Y (X_3 - \partial_y \Sigma)(|\varphi|^2 - |\varphi^c|^2) - |(\partial_y - g_Y Y \Sigma)\varphi|^2 - |(\partial_y + g_Y Y \Sigma)\varphi^c|^2 + \psi^c(\partial_y - g_Y Y \Sigma)\psi + \text{H.c.} \tag{3.8}$$

one obtains a supersymmetric mass term as in Eq. (3.3), with $M = -g_Y Y \xi/2$, where g_Y is the 5D hypercharge coupling.

Once more we are led to consider a mass term for the hypermultiplets. With a momentum cutoff Λ, the radiatively generated FI term is

$$\xi = \frac{g_Y}{16\pi^2}\Lambda^2 \tag{3.9}$$

which, in turn, translates itself into a mass term for a hypermultiplet of hypercharge Y

$$M_\xi(Y) = -Y \frac{g_Y^2}{16\pi^2}\frac{\Lambda^2}{2} = -Y \frac{g'^2 R}{16\pi}\Lambda^2 \tag{3.10}$$

where g' is the usual $U(1)$ coupling in 4D.

For $\Lambda R \simeq 5$ this is a small mass compared, for example, with the one-loop mass induced for the Higgs by the top loop (2.3). One can nevertheless consider an arbitrary value of ξ, as done in Sec. IV C. Finally it should be pointed out that this induced FI term has a geometric interpretation in 5D supergravity, suggesting that its renormalization vanishes beyond one loop and that, with a proper regularization, the case $\xi = 0$ is not inconceivable as coming from a suitable more fundamental theory [6].

D. Hypermultiplet spectrum in the presence of a mass term

It is useful to summarize how the hypermultiplet spectrum of Fig. 2 is modified in the presence of a mass term M as in Eq. (3.3). This spectrum is worked out in Appendix A both in the case of matter-like and Higgs-multiplet-like boundary conditions. The spectra in the two cases are shown in Figs. 4 and 5 respectively. A few things are useful to note. In the large MR limit a supersymmetric spectrum is restored with bound states localized at the boundaries. In Fig. 5 the lightest state which passes through zero at vanishing MR is the Higgs boson, with the cusp at $MR = 0$ reflecting the change of sign of the squared mass, which is $(m_h R)^2 \simeq (4/\pi) MR$ at $|MR| \ll 1$.

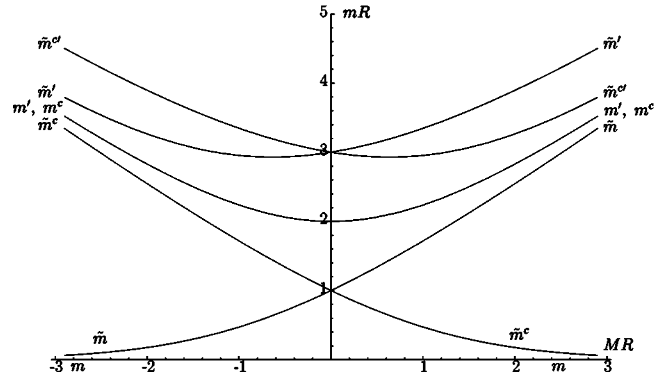

FIG. 4. Spectrum of a matter hypermultiplet, in units of $1/R$, as function of MR.

IV. ELECTROWEAK SYMMETRY BREAKING IN DETAIL

The purpose of this section is to study in detail the possible effects of hypermultiplet masses on EWSB. In this paper we consider the case that these masses do not exceed $1/R$, leaving the exploration of the alternative possibility to a future publication. As seen in Sec. III, moderate values of MR are consistent with radiative corrections. Other than the modification of the spectrum, hypermultiplet masses have 3 types of effects on the EWSB: (1) a mass term for the top Q or U hypermultiplets changes the relation between the top mass and the top-Higgs coupling, crucial in EWSB; (2) a mass term for the top Q or U hypermultiplets influences the one-loop Higgs mass or the complete one-loop effective potential; (3) a mass term for the Higgs hypermultiplet gives a tree level mass to the zero mode Higgs field (see Fig. 5), which feeds directly in the effective potential.

A. The top mass and the top Yukawa coupling

The relation between the top mass and the top-Higgs coupling y_t is obtained by solving the equation of motion for the lowest mode of the fermions in the top hypermultiplets Q and U, coupled by Eq. (2.1) at the $y = 0$ boundary. The in-

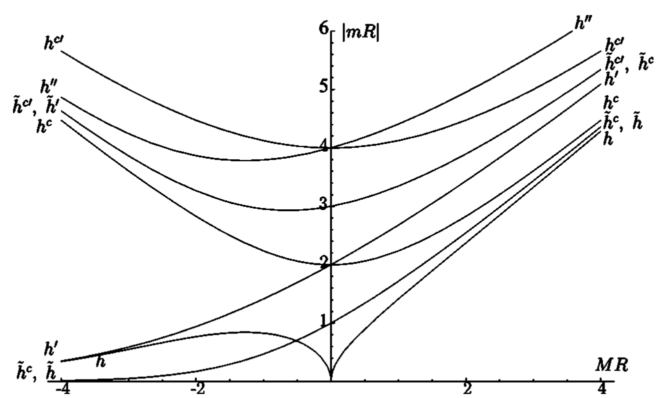

FIG. 5. As in Fig. 4 for the Higgs hypermultiplet.

teraction (2.1) is itself a localized mass term when the Higgs scalar is replaced by the vacuum expectation value v. This is done in Appendix A. The result can be expressed as

$$y_t = \hat{y}_t \eta_0^U \eta_0^Q \eta_0^h \tag{4.1}$$

where

$$\hat{y}_t = \frac{\lambda_t}{(2\pi R)^{3/2}} = \frac{2m_t}{\pi v} \frac{1}{\sqrt{\omega_-^U \omega_-^Q}} \tag{4.2}$$

$$\omega_\pm^i = k_i R \coth\left(\frac{k_i \pi R}{2}\right) \pm M_i R, \quad i = U,Q \tag{4.3}$$

$$k_i = (M_i^2 - m_t^2)^{1/2} \tag{4.4}$$

and $\eta_0^{U,Q,h}$ are the wave functions for the lightest U,Q,h modes at $y=0$, normalized to $\int_0^{2\pi R} dy |\eta^i(y)|^2 = 2\pi R$ and given in Appendix A. At $M_U = M_Q = M_h = 0$, Eq. (4.1) reduces to

$$y_t = \frac{m_t}{v} \frac{2\sin(\pi R m_t)}{\pi R m_t + \sin(\pi R m_t)}. \tag{4.5}$$

Note that in the limit $M_h = 0$, $|M_{U,Q}| \gg 1/R \gg m_t$, y_t reduces to the standard value, m_t/v, no matter what the sign of M is. For positive M, when the top wave function and the Yukawa coupling are localized at opposite boundaries, this is due to a compensating increase of $\hat{y}_t$, which is directly related to the fundamental coupling in the Lagrangian. When y_t and $\hat{y}_t$ differ significantly, it is $\hat{y}_t$ that enters into Eqs. (2.4), (2.5) to determine the point of saturation of perturbation theory.

B. One-loop Higgs effective potential for arbitrary M_U, M_Q

The one-loop Higgs mass (2.3) from the diagrams of Fig. 3 gets corrected by the presence of M_U and M_Q. This is in fact also true for the entire one-loop effective potential which has to be computed anyhow because of the large correction to the quartic coupling and because of the higher order terms in $(vR)^2$ which may be important insofar as R is not determined.

The calculation described in Sec. II immediately generalizes to the massive case in terms of the propagators in the presence of masses. Considering, as in Eq. (2.2), the mixed propagators at $y = y' = 0$ the effective potential due to top-quark–top-squark exchanges is

$$\begin{aligned} V_t(h;M_U,M_Q) = N_c \sum_{N=1}^{\infty} \int \frac{d^4p}{(2\pi)^4} \frac{(-1)^{N+1}}{N} \left(\frac{\lambda_t h\, \eta_0^h}{\sqrt{2\pi R}}\right)^{2N} &\times \{[G_\varphi^U(p,0) G_F^Q(p,0)]^N \\ &+ [G_\varphi^Q(p,0) G_F^U(p,0)]^N \\ &- 2[G_\psi^U(p,0) G_\psi^Q(p,0)]^N\} \end{aligned} \tag{4.6}$$

where $G_i^{U,Q}(p,y) = G_i(p,y;M_{U,Q})$ with $i = \phi,\ F,\ \psi$. The propagators $G_i(p,y;M)$ are given in Appendix B, while the wave function of the Higgs zero mode η_0^h is given in Appendix A. The integral is performed over the Euclidean 4-momentum.

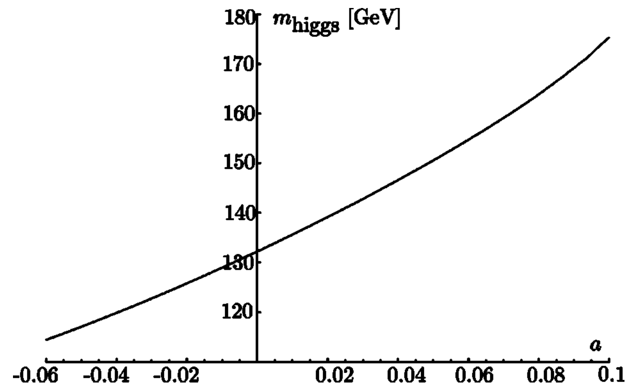

FIG. 6. Higgs mass as function of the dimensionless parameter a, Eq. (4.7).

C. Electroweak symmetry breaking in the presence of a FI term

As shown in Sec. III C, a FI term is equivalent for any hypermultiplet of hypercharge Y to a mass term, which we parametrize in terms of a dimensionless variable a as

$$M(Y) = \frac{a}{R} Y, \tag{4.7}$$

to be inserted in Eqs. (3.3), (3.4).

In the presence of these masses the potential we consider to determine the VEV of the Higgs field is

$$\begin{aligned} V(h;R,a) = m^2[M(1/2)]h^2 + \frac{21\xi(3)}{16\pi^4} \frac{g^2}{R^2} h^2 + \frac{g^2 + g'^2}{8} h^4 \\ + V_t(h; M(-2/3), M(1/6)). \end{aligned} \tag{4.8}$$

Other than the standard tree-level quartic coupling and the one-loop contribution from the top-quark–top-squark exchanges, Eq. (4.6), the potential includes the tree level mass $m(M)$ computed in Sec. III D and Appendix A [the first term on the right-hand side (rhs) of (4.8)] and a one-loop mass term from the KK tower of the $SU(2)$ gauge multiplets [second term on the rhs of Eq. (4.8) [15]].

Imposing the occurrence of the minimum at $h = v = 175$ GeV determines the Higgs boson mass m_h and $1/R$, together with the entire spectrum, as functions of a. The Higgs boson mass is shown in Fig. 6. The lightest top squarks, which are nondegenerate when $a \neq 0$ because $M_U \neq M_Q$, occurs in two chiralities. Their mass difference depends on the parameter a and is about 70 GeV at $a = 0.1$. The top squark masses together with $1/R$ are shown in Fig. 7. These figures refine those of Ref. [4].

The sharp increase of $1/R$ with a is due to an increasingly precise accidental cancellation (at about 10% level for $a = 0.1$) between the positive tree level squared mass in Eq. (4.8) and the negative contribution from the top-quark–top-

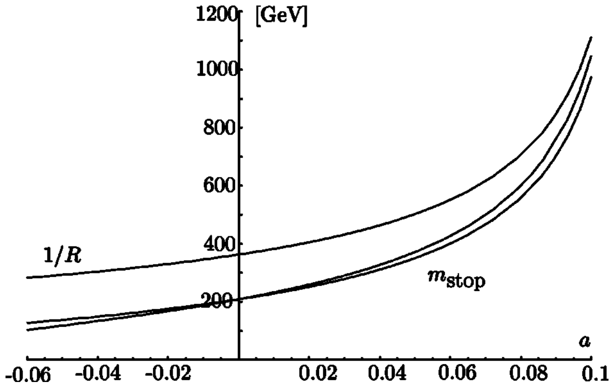

FIG. 7. Compactification scale and lightest top squark masses as functions of a, Eq. (4.7).

squark loop. Note that the estimate of the radiatively induced FI term in Eq. (3.10) corresponds to a negligibly small $a=10^{-2}$ [4].

Through R, M_U and M_Q, also the Higgs-boson–top quark coupling acquires a dependence on a, determined in Eq. (4.1) and shown in Fig. 8. Note that the top Yukawa coupling y_t is reduced from the standard model value by about 10% due to the localization of the interaction at the boundary.

D. Electroweak symmetry breaking with sizable $M_U = M_Q$

As we have seen, the mass terms from the FI term have to be small. Their effect can, however, be significant due to a possible cancellation occurring in the Higgs potential between the tree level Higgs squared mass and the radiatively induced effect. Here we consider the possible effects of direct masses for the U,Q hypermultiplets, taking $M_U = M_Q = M$ for simplicity. At the same time, again for simplicity, we set the FI term, or the a parameter, to zero.

Proceeding as in Sec. IV C the Higgs potential we consider is

$$V(h;R,M)=\frac{21\xi(3)}{16\pi^4}\frac{g^2}{R^2}h^2+\frac{g^2+g'^2}{8}h^4+V_t(h;M,M) \tag{4.9}$$

whose minimization determines m_h and R as functions of M. The Higgs boson mass is shown in Fig. 9 for $-0.4\leqslant MR\leqslant 0.1$.

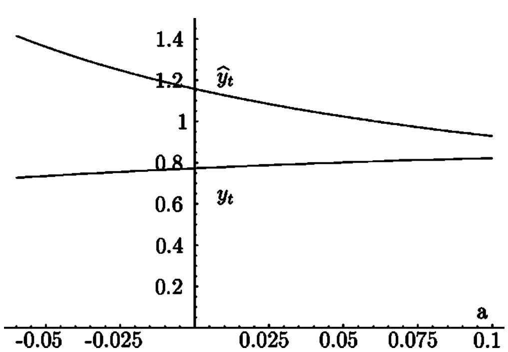

FIG. 8. Top-quark–Higgs-boson coupling (y_t) and $\hat{y}_t = \lambda_t/(2\pi R)^{3/2}$ as functions of a, Eq. (4.7).

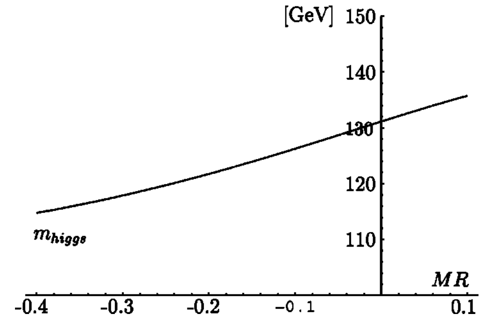

FIG. 9. Higgs mass as function of MR.

The reason for interrupting MR below 0.1 is that the mass of lightest top squarks [21] falls below the experimental lower bound of about 150 GeV, as shown in Fig. 10. For MR below -0.4, instead, it is the Higgs boson which becomes too light. This result, however, could not persist for $MR<-1$, where higher-loop gauge corrections become important [16]. This case will be analyzed elsewhere. In the interval $-0.4\leqslant MR\leqslant 0.1$, both $1/R$ and $m_{\tilde{t}}$ have a non-negligible dependence on M, as shown in Fig. 10. The degeneracy between the two lightest top squark masses would be resolved by taking $M_U \neq M_Q$. The top-squark–Higgs-boson couplings y_t and $\hat{y}_t$ in this case are shown in Fig. 11.

V. SPECTRUM AND PHENOMENOLOGICAL IMPLICATIONS

In the absence of hypermultiplet mass terms, the value of the compactification scale and the spectrum of the lightest particles is given in Table II with an error that estimates the uncertainties due to the presence of the extra couplings and operators mentioned in Sec. II A [2]. By letting the mass terms vary in a moderate range, very consistent with radiative corrections, the main deviation from the massless case is due to a possible mass term for the Higgs hypermultiplet which can partially counteract the top-quark–top-squark radiative corrections that trigger EWSB. This can in turn drive up the compactification scale and, consequently, the entire spectrum.

In Sec. IV C we have explicitly discussed the effects of a FI term, which is a particular example of this case. The entire spectrum becomes therefore effectively determined by $1/R$ in the range of Fig. 7, 300 GeV $\lesssim R^{-1}\lesssim 1000$ GeV. The de-

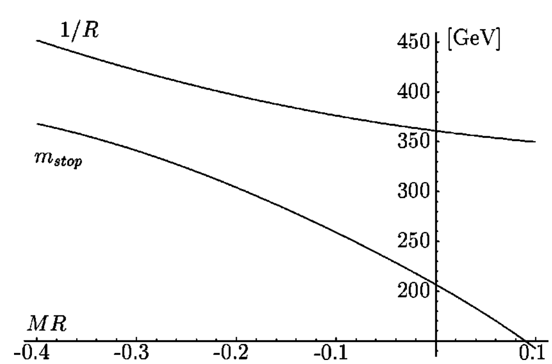

FIG. 10. Top squark mass and 1/R as functions of MR.

TABLE II. The particle spectrum and $1/R$ in the absence of any mass term (Column A) and in the presence of a FI term (Column B). All entries are in GeV.

	A	B
$1/R$	360 ± 70	$300\div 1000$
h	133 ± 10	Fig. 12
$\tilde t_1,\ \tilde u_1$	210 ± 20	$1/R(1\pm 8\%)-m_t$
$\chi^{\pm},\ \chi^0,$ $\tilde g,\ \tilde q,\ \tilde l$	360 ± 70	$1/R(1\pm 20\%)$
$\tilde t_2,\ \tilde u_2$	540 ± 30	$1/R(1\pm 8\%)+m_t$
A_1,q_1,l_1,h_1	720 ± 140	$2/R(1\pm 20\%)$

pendence of m_h on $1/R$ is shown in Fig. 12 obtained from Figs. 6 and 7, whereas the masses of the other particles are again given in Table II. Note that the lightest top squark $\tilde t_1$ is the lightest supersymmetric particle (LSP), except possibly for large values of $1/R$ where the corrections due to kinetic terms localized on the boundaries, giving rise to the main uncertainty indicated in Table II, could reverse the order with any of the other superpartners at $1/R$. Unless an explicit violation of the $U(1)_R$–symmetry were introduced at the boundaries, the LSP would be stable. A moderate effect could also arise from an explicit mass term for the top hypermultiplets, as shown in Fig. 10.

A. Phenomenological implications

Except for the large, somewhat fine-tuned values of $1/R$, the Higgs boson is below the WW threshold, with a preferred mass in the 130 GeV range. It has SM-like couplings to $b\bar b$ and $\tau\bar\tau$ and WWh, ZZh gauge couplings. It could therefore be looked at in an associated production of Wh or Zh, followed by $b\bar b$ and $\tau\bar\tau$ decays. We have already mentioned the deviation of the top Yukawa coupling from the SM value (see Figs. 8, 11). More important for the possible discovery in a hadron collider is the suppression of the Higgs–gluon–gluon squared coupling, ranging from 10% to 60% relative to the SM value as $1/R$ increases from 300 to about 700 GeV, where the WW threshold is crossed [17].

A main feature of the model is that the two degenerate light top squark are the LSP and are stable if $U(1)_R$ is exact.

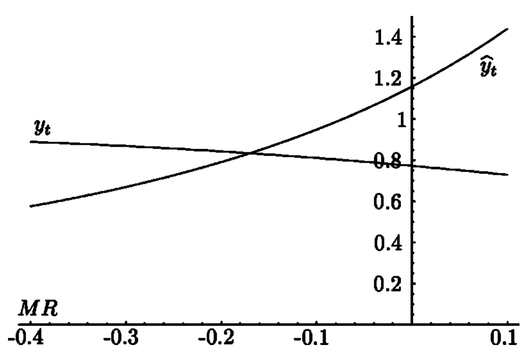

FIG. 11. Top-quark–Higgs-boson coupling y_t and $\hat y_t = \lambda_t/(2\pi R)^{3/2}$ as functions of MR.

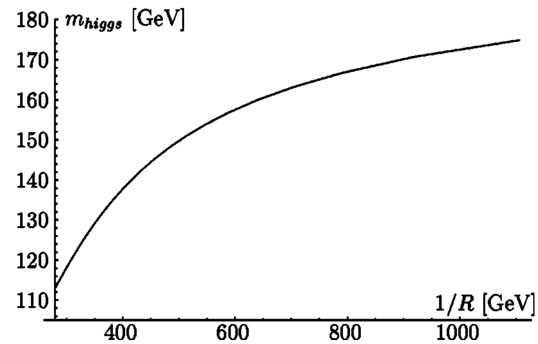

FIG. 12. Higgs boson mass versus $1/R$ in presence of a Fayet-Iliopoulos term.

Their mass is approximately $(1/R-m_t)$ with a lowest preferred value in the 200 GeV range. In such a low range value the super-hadrons $T^+=\tilde t_1\bar d$ and $T^0=\tilde t_1\bar u$ and their charge conjugates $T^-,\bar T^0$ could easily be detected at the Tevatron run II as stable particles, since their possible decay into one another is slow enough to let them both cross the detector. $T^{\pm}$ could appear as a stiff charge track with little hadron calorimeter activity, hitting the muon chambers and distinguishable from a muon via dE/dx and time-of-flight. The neutral states, on the contrary, could be identified as missing energy since they could traverse the detector with little interaction. The cross section for the pair production at the Tevatron of the top squarks with a 200 GeV mass is 0.6 pb [18].

The heavier supersymmetric particles in Table II could be looked at through their chain decay into the LSP. Similarly the discovery of the first states at $2/R$ of the KK tower of SM particles (heavy quarks, leptons with their mirror partners, heavy gauge and Higgs bosons) would be strong evidence for the picture of EWSB described in this paper. Note that (discrete) momentum conservation in the 5th dimension forbids unsuppressed gauge couplings of the heavy gauge bosons to the standard fermions.

ACKNOWLEDGMENTS

We thank Roberto Contino, Paolo Creminelli, Lawrence Hall, Federico Minneci, Takemichi Okui, Steven Oliver, and Riccardo Rattazzi for useful discussions. This work has been partially supported by the EC under TMR contract HPRN-CT-2000-00148.

APPENDIX A: SPECTRUM

In this appendix we calculate the KK spectrum of Higgs and matter hypermultiplets in the presence of a mass term M as in Eq. (3.3).

Let (φ,ψ,F), (φ^c,ψ^c,F^c) be either a Higgs or a matter hypermultiplet. In the presence of a mass term as in the (3.3) the Lagrangian upon eliminating the F-terms is

$$\mathcal{L}=|\partial_M\varphi|^2+|\partial_M\varphi^c|^2+i\,\psi\sigma^\mu\partial_\mu\bar\psi+i\,\psi^c\sigma^\mu\partial_\mu\bar\psi^c+\psi^c\partial_y\psi + \bar\psi^c\partial_y\bar\psi-M^2(|\varphi|^2+|\varphi^c|^2)-2M\left[\delta(y)+\delta\left(y-\frac{\pi R}{2}\right)\right] \times(|\varphi|^2-|\varphi^c|^2)-M\,\eta(y)(\psi\psi^c+\overline{\psi\,\psi^c}) \tag{A1}$$

where $M=0,1,2,3,5$ while $\mu=0,1,2,3$.

Thus the equations of motion are

$$\left\{\partial_M\partial^M+M^2+2M\left[\delta(y)+\delta\left(y-\frac{\pi R}{2}\right)\right]\right\}\varphi=0 \quad \text{(A2a)}$$

$$\left\{\partial_M\partial^M+M^2-2M\left[\delta(y)+\delta\left(y-\frac{\pi R}{2}\right)\right]\right\}\varphi^c=0 \quad \text{(A2b)}$$

$$\left\{\partial_M\partial^M+M^2+2M\left[\delta(y)-\delta\left(y-\frac{\pi R}{2}\right)\right]\right\}\psi=0 \quad \text{(A2c)}$$

$$\left\{\partial_M\partial^M+M^2-2M\left[\delta(y)-\delta\left(y-\frac{\pi R}{2}\right)\right]\right\}\psi^c=0. \quad \text{(A2d)}$$

Equations (A2) must be solved imposing the proper boundary conditions to the fields φ, φ^c, ψ, ψ^c. Note that the delta functions on the left-hand side of Eqs. (A2) are both present only if the field under consideration has $(+,+)$ parity under $Z_2\times Z_2'$ symmetry. In all other cases the wave function vanishes at $y=0$ and/or $y=\pi R/2$ and the delta functions in the corresponding points are irrelevant.

1. Matter hypermultiplets

If we consider a matter hypermultiplet, then Eqs. (A2) become

$$\left\{\partial_M\partial^M+M^2+2M\left[\delta(y)-\delta\left(y-\frac{\pi R}{2}\right)\right]\right\}\psi=0 \quad \text{(A3a)}$$

$$[\partial_M\partial^M+M^2]\psi^c=0 \quad \text{(A3b)}$$

$$[\partial_M\partial^M+M^2+2M\,\delta(y)]\varphi=0 \quad \text{(A3c)}$$

$$\left[\partial_M\partial^M+M^2-2M\,\delta\left(y-\frac{\pi R}{2}\right)\right]\varphi^c=0. \quad \text{(A3d)}$$

Taking for the wave functions the following form:

$$\psi(x,y)=\begin{cases}\tilde{\psi}(x)\left[A_\psi\sin k\left(y-\frac{\pi R}{2}\right)+B_\psi\cos k\left(y-\frac{\pi R}{2}\right)\right], & y\in\left(0,\frac{\pi R}{2}\right)\\ \tilde{\psi}(x)\left[-A_\psi\sin k\left(y-\frac{\pi R}{2}\right)+B_\psi\cos k\left(y-\frac{\pi R}{2}\right)\right], & y\in\left(\frac{\pi R}{2},\pi R\right)\end{cases}$$

$$\psi^c(x,y)=\tilde{\psi}^c(x)A_{\psi^c}\sin ky, \quad y\in(0,\pi R)$$

$$\varphi(x,y)=\tilde{\varphi}(x)A_\varphi\sin k\left(y-\frac{\pi R}{2}\right), \quad y\in(0,\pi R)$$

$$\varphi^c(x,y)=\tilde{\varphi}^c(x)\begin{cases}A_{\varphi^c}\sin ky, & y\in\left(0,\frac{\pi R}{2}\right)\\ B_{\varphi^c}\sin k(\pi R-y), & y\in\left(\frac{\pi R}{2},\pi R\right),\end{cases} \quad \text{(A4)}$$

the mass of every field is given by $m^2=M^2+k^2$ where k is constrained by Eqs. (A3). Imposing the proper conditions on the wave functions and their first derivatives on the boundary we get the following equations for k:

$$\psi(+,+)\Rightarrow(k^2+M^2)\sin\frac{k\pi R}{2}=0 \quad \text{(A5a)}$$

$$\psi^c(-,-)\Rightarrow\sin\frac{k\pi R}{2}=0 \quad \text{(A5b)}$$

$$\varphi(+,-)\Rightarrow\tan\frac{k\pi R}{2}=-\frac{k}{M} \quad \text{(A5c)}$$

$$\varphi^c(-,+)\Rightarrow\tan\frac{k\pi R}{2}=\frac{k}{M}. \quad \text{(A5d)}$$

A few things are worth noticing:

(1) One can get the equations for the bound states by analytical continuation, setting $k=i\rho$ in Eq. (A5).

(2) The bound state $\psi(+,+)$ is massless for every value of M, while the excited states have masses $(m_\psi^2)_n=M^2+(2n/R)^2$, $n=1,2,\ldots$.

(3) The equation for $\psi^c(-,-)$ is unaffected by the presence of M because of the vanishing of the wave function at $y=0$, $\pi R/2$.

2. Higgs hypermultiplet

If we consider a Higgs hypermultiplet, then Eqs. (A2) become

$$\left\{\partial_M\partial^M+M^2+2M\left[\delta(y)+\delta\left(y-\frac{\pi R}{2}\right)\right]\right\}h=0 \quad \text{(A6a)}$$

$$[\partial_M \partial^M + M^2] h^c = 0 \tag{A6b}$$

$$[\partial_M \partial^M + M^2 + 2M\,\delta(y)]\lambda = 0 \tag{A6c}$$

$$\left[\partial_M \partial^M + M^2 + 2M\,\delta\left(y - \frac{\pi R}{2}\right)\right]\lambda^c = 0. \tag{A6d}$$

With the same procedure of the matter case one gets the equations

$$h(+,+) \Rightarrow \tan\frac{k\pi R}{2} = \frac{2kM}{k^2 - M^2} \tag{A7a}$$

$$h^c(-,-) \Rightarrow \sin\frac{k\pi R}{2} = 0 \tag{A7b}$$

$$\lambda(+,-) \Rightarrow \tan\frac{k\pi R}{2} = -\frac{k}{M} \tag{A7c}$$

$$\lambda^c(-,+) \Rightarrow \tan\frac{k\pi R}{2} = -\frac{k}{M}. \tag{A7d}$$

Note that Eq. (A7a) for the bound state of the h field leads to a negative squared mass if $M<0$.

Solving Eq. (A6a) for the zero mode of the Higgs scalar we find for the wave function normalized to $\int_0^{2\pi R} dy |h^{(0)}(y)|^2 = 1$

$$h^{(0)}(y) = \frac{-k\cos k\left(y - \frac{\pi R}{2}\right) + M\sin k\left(y - \frac{\pi R}{2}\right)}{\sqrt{2M + (k^2 + M^2)\pi R - 2M\cos k\pi R + \left(k - \frac{M^2}{k}\right)\sin k\pi R}} \tag{A8}$$

with k the solution of Eq. (A7a). Expression (A8) is valid for $y \in [0, \pi R/2)$. Note that if $M \to 0$, then $h^{(0)}(y) \to (2\pi R)^{-1/2}$.

3. Spectrum in presence of a VEV for the Higgs field

If we have a top quark hypermultiplet, then the top Yukawa coupling [22]

$$\mathcal{L}_Y = \frac{\lambda_t}{2}[\delta(y) + \delta(y - \pi R)] \int d^2\theta \hat{h}\,\hat{Q}\,\hat{U} + \text{H.c.} \tag{A9}$$

leads to a mass term when we replace the Higgs zero mode $h^{(0)}$ with its VEV v. To calculate the spectrum in the presence of such a term it is convenient to rewrite the Lagrangian (A1) without eliminating the F auxiliary fields and use the following vectors:

$$X = \begin{pmatrix} \varphi \\ F^{c\dagger} \end{pmatrix}, \quad Y = \begin{pmatrix} \varphi^c \\ F^\dagger \end{pmatrix}, \quad Z = \begin{pmatrix} \psi \\ \overline{\psi^c} \end{pmatrix}. \tag{A10}$$

Then Eq. (A1) becomes

$$\mathcal{L} = (X^\dagger_{U,Q} Y^\dagger_{Q,U}) M_B \begin{pmatrix} X_{U,Q} \\ Y_{Q,U} \end{pmatrix} + (\overline{Z}_{U,Q} Z^t_{Q,U}) M_F \begin{pmatrix} Z_{U,Q} \\ \overline{Z}^t_{Q,U} \end{pmatrix} \tag{A11}$$

where

$$M_B = \begin{pmatrix} -\Box - 4M_{U,Q}\delta_{\pi R/2} & \partial_y + \eta(y) M_{U,Q} & 0 & \lambda_t \alpha^* \\ -\partial_y + \eta(y) M_{U,Q} & 1 & 0 & 0 \\ 0 & 0 & -\Box + 4M_{Q,U}\delta_{\pi R/2} & -\partial_y + \eta(y) M_{Q,U} \\ \lambda_t \alpha & 0 & \partial_y + \eta(y) M_{Q,U} & 1 \end{pmatrix} \tag{A12a}$$

$$M_F = \begin{pmatrix} \partial_y - \eta(y) M_{U,Q} & i\sigma^\mu \partial_\mu & 0 & 0 \\ i\overline{\sigma}^\mu \partial_\mu & -\partial_y - \eta(y) M_{U,Q} & 0 & \lambda_t \alpha^* \\ \lambda_t \alpha & 0 & -\partial_y - \eta(y) M_{Q,U} & i\sigma^\mu \partial_\mu \\ 0 & 0 & i\overline{\sigma}^\mu \partial_\mu & \partial_y - \eta(y) M_{Q,U} \end{pmatrix} \tag{A12b}$$

$$\alpha = \frac{1}{2}[\delta(y) + \delta(y - \pi R)]v\,h^{(0)}(y=0), \qquad \delta_{\pi R/2} = \delta\left(y - \frac{\pi R}{2}\right)$$

with $h^{(0)}(y=0)$ the Higgs zero mode wave function (A8) at $y=0$.

Taking for the wave functions the form (A4) (in this case one must consider also the wave functions of $F_{U,Q}, F^c_{Q,U}$) and imposing the proper boundary conditions one can get the equations for the masses of the fields $\varphi_{U,Q}, \psi_{U,Q}, \varphi^c_{U,Q}, \psi^c_{U,Q}$.

For the top quark $\psi_{U,Q}$ and the top squark $\varphi_{U,Q}$ (the lowest modes) one gets

$$m_t^2=\frac{\lambda_t^2 v^2|h^{(0)}(0)|^2}{16}\left(k_U^t\coth\frac{k_U^t\pi R}{2}-M_U\right)\times\left(k_Q^t\coth\frac{k_Q^t\pi R}{2}-M_Q\right) \tag{A13}$$

$$\left(k_U^{\tilde t}\coth\frac{k_U^{\tilde t}\pi R}{2}+M_U\right)\left(k_Q^{\tilde t}\coth\frac{k_Q^{\tilde t}\pi R}{2}-M_Q\right)=\frac{\lambda_t^2 v^2|h^{(0)}(0)|^2}{16}\left[m_{\tilde t}^2+2M_Q\left(k_Q^{\tilde t}\coth\frac{k_Q^{\tilde t}\pi R}{2}-M_Q\right)\right] \tag{A14}$$

where $k_{U,Q}^{t,\tilde t}=\sqrt{M_{U,Q}^2-m_{t,\tilde t}^2}$. The wave function of the top quark zero mode, normalized to $\int_0^{2\pi R}dy|\psi_0^{U,Q}(y)|^2=1$, is obtained by solving Eq. (A3a). For $y\in[0,\pi R/2)$ we have

$$\psi_0^{U,Q}(y)=\frac{k_{U,Q}\cosh k_{U,Q}\left(\frac{\pi R}{2}-y\right)-M_{U,Q}\sinh k_{U,Q}\left(\frac{\pi R}{2}-y\right)}{\sqrt{-m_t^2\pi R+2M_{U,Q}(-\cosh k_{U,Q}\pi R+1)+[k_{U,Q}+(M^2/k_{U,Q})]\sinh k_{U,Q}\pi R}}. \tag{A15}$$

The usual 4D top Yukawa coupling, defined as the coupling among the zero modes of the fields h,ψ_U,ψ_Q, is given by

$$y_t=\hat y_t\eta_0^h\eta_0^U\eta_0^Q \tag{A16}$$

where $\hat y_t=\lambda_t(2\pi R)^{-3/2}$ and η_0^i $(i=h,U,Q)$ are the wave functions (A8)–(A15) at $y=0$ normalized in such a way that

$$\int_0^{2\pi R}dy|\eta_0^i(y)|^2=2\pi R. \tag{A17}$$

Inserting Eq. (A16) into Eq. (A13) we obtain the relation between the top quark mass m_t and the 4D Yukawa coupling y_t.

APPENDIX B: PROPAGATORS

In order to obtain the mixed momentum–coordinate space propagators for the components φ, ψ, F of a hypermultiplet, we start from the 5D Lagrangian without eliminating the auxiliary fields. If $M\neq 0$ the relevant part of this Lagrangian is

$$\begin{aligned}\mathcal{L}=&|\partial_\mu\varphi|^2+|\partial_\mu\varphi^c|^2+|F|^2+|F^c|^2+i\overline{\psi\sigma}^\mu\partial_\mu\psi+i\psi^c\sigma^\mu\partial_\mu\overline{\psi}^c\\&+[F\partial_y\varphi^c-F^c\partial_y\varphi+\text{H.c.}]+M\eta(y)[F\varphi^c+F^c\varphi+\text{H.c.}]\\&+[\psi^c\partial_y\psi+\text{H.c.}]-M\eta(y)[\psi^c\psi+\text{H.c.}]\\&-4M\delta(y-\pi R/2)[|h|^2-|h^c|^2]\end{aligned} \tag{B1}$$

for the generic hypermultiplet of components φ, ψ, F and their conjugates. The border term at $\pi R/2$ in the last line is necessary to maintain 5D supersymmetric invariance under ξ^{-+} transformations [23]. In this paper we need only the propagators for the top-quark–top-squark sector, so we can assume from now on that the parities are those of the matter multiplets. Using the vectors defined in Eq. (A10), $\mathcal{L}$ can be recast in a more compact form as in Appendix A:

$$\mathcal{L}=X^\dagger AX+Y^\dagger BY+\overline{Z}CZ \tag{B2}$$

where

$$A=\begin{pmatrix}-\Box-4M\delta(y-\pi R/2) & \partial_y+M\eta(y)\\ -\partial_y+M\eta(y) & 1\end{pmatrix},$$

$$B=\begin{pmatrix}-\Box+4M\delta(y-\pi R/2) & -\partial_y+M\eta(y)\\ \partial_y+M\eta(y) & 1\end{pmatrix},$$

$$C=\begin{pmatrix}\partial_y-M\eta(y) & i\sigma^\mu\partial_\mu\\ i\overline{\sigma}^\mu\partial_\mu & -\partial_y-M\eta(y)\end{pmatrix}. \tag{B3}$$

Note that the components of X (or Y, Z) have the same quantum numbers but different boundary conditions.

Let us focus, for example, on the propagator

$$G[(x-x')_\mu;y,y']=\langle\varphi(x'_\mu,y')\varphi^\dagger(x_\mu,y)\rangle, \tag{B4}$$

the others being analogous. In general all the correlation functions will depend on both y and y' [24] because of the non-conservation of the 5th component of momentum in the segment $[0,\pi R/2]$. However, being interested in calculating only loops formed using Yukawa interactions which are localized at $y=0$, we can impose without any problem $y'=0$ from the very beginning of the calculation, reducing the dependence of Eq. (B4) only to $(x-x')_\mu$ and y.

One can arrange propagators in matrices using the vectors previously defined. In particular, defining

$$\mathcal{G}(x-x';y)=\langle X(x,y)X^{\dagger}(x',0)\rangle =\begin{pmatrix} \langle \varphi(x,y)\varphi^{\dagger}(x',0)\rangle & \langle \varphi(x,y)F^{c}(x',0)\rangle \\ \langle F^{c\dagger}(x,y)\varphi^{\dagger}(x',0)\rangle & \langle F^{c\dagger}(x,y)F^{c}(x',0)\rangle \end{pmatrix} \tag{B5}$$

the equations of motion for the scalar Green functions are

$$A\mathcal{G}(x-x';y)=i\sigma_3\delta^{(4)}(x-x')\frac{1}{2}[\delta(y)+\delta(y-\pi R)] \tag{B6}$$

where σ_3 is the usual Pauli matrix. Multiplying the 1st row of A by the 1st column of $\mathcal{G}$ we get a system of 2 differential equations, which after passing to the Euclidian 4-momentum, assumes the form:

$$-k_4^2 g(y)-4M\delta(y-\pi R/2)+[\partial_y+M\eta(y)]f(y)=i\delta(y)/2$$

$$[-\partial_y+M\eta(y)]g(y)+f(y)=0$$

where $g(y)=\langle\varphi(y)\varphi^{\dagger}(0)\rangle$ and $f(y)=\langle F^{c\dagger}(y)\varphi^{\dagger}(0)\rangle$. These coupled equations must be solved imposing the $(+,-)$ and the $(-,+)$ boundary conditions in y on g and f, respectively, using the same techniques of Appendix A. One finally gets the $\langle\varphi\varphi^{\dagger}\rangle$ propagator $G_{\varphi}(k_4,y;M)$:

$$G_{\varphi}(k_4,y;M)=\langle\varphi\varphi^{\dagger}\rangle(y) =\frac{-i\sinh\left[k\left(\frac{\pi R}{2}-y\right)\right]}{4\left[k\cosh\left(\frac{k\pi R}{2}\right)+M\sinh\left(\frac{k\pi R}{2}\right)\right]} \tag{B7}$$

where $k=\sqrt{k_4^2+M^2}$. Analogously the $\langle FF^{\dagger}\rangle$ $(+,-)$ and $\langle\psi\psi^{\dagger}\rangle$ $(+,+)$ propagators are

$$G_F(k_4,y;M)=\frac{i\,k_4^2}{4}\frac{\sinh\left[k\left(\frac{\pi R}{2}-y\right)\right]}{\left[k\cosh\left(\frac{k\pi R}{2}\right)-M\sinh\left(\frac{k\pi R}{2}\right)\right]}\times\left\{1-\frac{2M}{k_4^2}\left[k\coth\left(\frac{k\pi R}{2}\right)-M\right]\right\}$$

$$G_{\psi}(k_4,y;M) =\frac{-i\,\not{k}_4}{4k_4^2}\frac{k\cosh\left[k\left(\frac{\pi R}{2}-y\right)\right]-M\sinh\left[k\left(\frac{\pi R}{2}-y\right)\right]}{\sinh\left(\frac{k\pi R}{2}\right)} \tag{B8}$$

where $\not{k}_4=\sigma\cdot k_4$.

In the limit $M\to 0$ these propagators become

$$G_{\varphi}(k_4,y;M=0)=\frac{-i}{4k_4}\frac{\sinh\left[k_4\left(\frac{\pi R}{2}-y\right)\right]}{\cosh\left(\frac{k_4\pi R}{2}\right)}$$

$$G_F(k_4,y;M=0)=\frac{i\,k_4}{4}\frac{\sinh\left[k_4\left(\frac{\pi R}{2}-y\right)\right]}{\cosh\left(\frac{k_4\pi R}{2}\right)}$$

$$G_{\psi}(k_4,y;M=0)=\frac{-i\,\not{k}_4}{4k_4}\frac{\cosh\left[k_4\left(\frac{\pi R}{2}-y\right)\right]}{\sinh\left(\frac{k_4\pi R}{2}\right)}. \tag{B9}$$

[1] R. Barbieri and A. Strumia, hep-ph/0007265.
[2] R. Barbieri, L.J. Hall, and Y. Nomura, Phys. Rev. D **63**, 105007 (2001).
[3] D.M. Ghilencea, S. Groot Nibbelink, and H.P. Nilles, Nucl. Phys. **B619**, 385 (2001).
[4] R. Barbieri, L.J. Hall, and Y. Nomura, hep-ph/0110102.
[5] C.A. Scrucca, M. Serone, L. Silvestrini, and F. Zwirner, Phys. Lett. B **525**, 169 (2002).
[6] R. Barbieri, R. Contino, P. Creminelli, R. Rattazzi, and C.A. Scrucca, Phys. Rev. D **66**, 024025 (2002).
[7] N. Arkani-Hamed, L.J. Hall, Y. Nomura, D.R. Smith, and N. Weiner, Nucl. Phys. **B605**, 81 (2001).
[8] Z. Chacko, M.A. Luty, and E. Ponton, J. High Energy Phys. **07**, 036 (2000).
[9] R. Barbieri, L.J. Hall, and Y. Nomura, Nucl. Phys. **B624**, 63 (2002).
[10] N. Arkani-Hamed, A.G. Cohen, and H. Georgi, Phys. Lett. B **516**, 395 (2001).
[11] L. Pilo and A. Riotto, hep-th/0202144.
[12] R. Barbieri, S. Ferrara, D.V. Nanopoulos, and K.S. Stelle, Phys. Lett. **113B**, 219 (1982).
[13] S. Ferrara, L. Girardello, T. Kugo, and A. Van Proeyen, Nucl. Phys. **B223**, 191 (1983).
[14] E.A. Mirabelli and M.E. Peskin, Phys. Rev. D **58**, 065002 (1998).
[15] I. Antoniadis, S. Dimopoulos, A. Pomarol, and M. Quiros, Nucl. Phys. **B544**, 503 (1999).
[16] D. Marti and A. Pomarol, hep-ph/0205034.
[17] G. Cacciapaglia, M. Cirelli, and G. Cristadoro, Phys. Lett. B **531**, 105 (2002).
[18] M. Chertok, G.D. Kribs, Y. Nomura, W. Orejudos, B. Schumm, and S. Su, in Proceedings of the APS/DPF/DPB Summer

Study on the Future of Particle Physics (Snowmass, 2001), edited by R. Davidson and C. Quigg, hep-ph/0112001.
[19] N. Arkani-Hamed, T. Gregoire, and J. Wacker, J. High Energy Phys. **03**, 055 (2002).
[20] A factor of 2 is included to account for the coupling of the nonzero KK modes, $\sqrt{2}$ times stronger than for the zero mode.
[21] The lightest top squark now comes in two degenerate chiralities.
[22] Here we are working on the interval $(0,2\pi R)$.
[23] Using the formulation in terms of 4D $N=1$ superfields [19] one privileges only one of the two local supersymmetries, here ξ^{+-}. This explains the apparent asymmetry between $y=0$ and $y=\pi R/2$.
[24] Or on $\overline{y}=(y+y')/2$ and $\Delta y=(y-y')$.